ONE DIMENSIONAL STABILITY OF NORMAL COMBUSTION OF GASES IN A CONSTANT MAGNETIC FIELD

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UDC 536.46:538.6

Investigation results are presented of one dimensional disturbances of normal flame of electrically conductive gas in a magnetic field; higher combustion stability is established due to a stabilizing effect of the field.

In [1] one dimensional stability was investigated of gas combustion in which the effect has been taken into account of the internal structure of the flame on the combustion process. It was shown that in this case the compressibility of gases has a stabilizing effect on the combustion process. The effect of magnetic field on the combustion of electrically conductive gas gives rise to specific magneto-hydrodynamic effects which are not only related to the internal flame structure but also to the intensity of the magnetic field.

In this article the one dimensional combustion stability is therefore considered by taking into account the compressibility as well as the effect of the magnetic field. Similarly as in [1] the internal structure of the flame is also considered (there is only one dimension in the problem, namely the width L of the flame zone).

Let a stationary two dimensional flame of an ideal gas be contained between the planes $x = -L$ and $x = 0$. A constant magnetic field is applied parallel to these planes. The gas moving in the positive direction of the x-axis passes through three regions: 1) the region of the reference hot mixture ($x \le -L$); 2) the region of the combustion products ($x \ge 0$); 3) the combustion region ($-L \le x \le 0$). In our subsequent considerations all the flow parameters carry subscripts depending on these regions. Average quantities over its width are adopted as constant parameters in the 3rd region.

In the 1st region the gas is not electrically conductive therefore the magnetic field has no effect on the gas.

In the region of the reaction products and the combustion zone (zones 2 and 3) the gas becomes electrically conductive if heated to high temperatures. In these zones the effect of magnetic field on the gas glow is considerable.

If for any reason the flame deflects from its original position by a quantity $\varepsilon = B \exp \omega t$ it will cause a disturbance in the entire gas in the form of acoustic (p'_{jk}, v'_{jk}), magnetoacoustic (h'_{jk}) and entropy waves (S'_j).

The linearized system of equations of magneto-gas dynamics in the regions 2 and 3 is as follows:

$$\begin{gathered}
\frac{\partial \rho_j'}{\partial t} + U_j \frac{\partial \rho_j'}{\partial x} + \rho_j \frac{\partial v_{jk}'}{\partial x} = 0, \\
\rho_1 \frac{\partial v_{jk}'}{\partial t} + \rho_j U_j \frac{\partial v_{jk}'}{\partial x} + \frac{1}{4\pi} H_0 \frac{\partial h_{jk}'}{\partial x} + \frac{\partial p_{jk}'}{\partial x} = 0, \\
H_0 \frac{\partial v_{jk}'}{\partial x} + U_j \frac{\partial h_{jk}'}{\partial x} + \frac{\partial h_{jk}'}{\partial t} = 0, \\
\frac{\partial S_j'}{\partial t} + U_j \frac{\partial S_j'}{\partial x} = 0.
\end{gathered} \tag{1}$$

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 20, No. 5, pp. 841-845, May, 1971. Original article submitted June 9, 1970.

A solution of this system is

$$v'_{jk} = A_{jk} \exp\{\gamma_{jk}(x+L)+\omega t\} \equiv A_{jk}\Psi_{jk},$$

$$\gamma_{jk} = -\frac{\omega}{U_j}\cdot\frac{M_j}{M_j+(-1)^k\sqrt{1+M_{mj}^2}},\quad h'_{jk} = -A_{jk}\frac{H_0\gamma_{jk}}{\omega+U_j\gamma_{jk}},$$

$$\frac{p'_{jk}}{\rho_j U_j} = -A_{jk}\Psi_{jk}\left[1+\frac{\omega}{\gamma_{jk}U_j}-\frac{M_{mj}^2}{M_j^2}\cdot\frac{1}{1+\frac{\omega}{\gamma_{jk}U_j}}\right],$$

$$S'_j = D_j\Psi_{j3},$$

where

$$\Psi_{j3} = -\frac{\omega}{U_j};\quad M_j = \frac{U_j}{c_j};\quad M_{mj}^2 = \frac{H_0^2}{4\pi\rho_j c_j^2}\ (j=2,3;\ k=1,2).$$

For the 1st region one should consider the standard system of equations of gas dynamics. Its solution is similar to that given above, the only difference being as follows: all the quantities M_{mj} must be set equal to zero.

The combustion disturbance takes place in the combustion region itself; therefore in the region j = 1 acoustic waves are admitted which escape upwards (k = 1), and in the region j = 2 one admits acoustic waves and disturbances of magnetic field which escape downwards (k = 2).

A reverse effect on the combustion process of the disturbances takes place by the interaction of the acoustic wave (p'_{jk}, v'_{jk}) and the internal structure of the combustion region j = 3; it is described by the feedback equation obtained in [1]. Here the effect of the magnetic field is taken into account via the equation of electromagnetic induction. The feedback equation is given by

$$v'_1|_{x=-L} - \frac{d\varepsilon}{dt} = qU_1\int_{t-\tau}^{t}\left.\frac{\partial v'_3}{\partial x}\right|_{x=U_3(t'-t)} dt'. \tag{2}$$

In the above τ denotes the characteristic combustion time; $q = U_3/U_1 = \alpha-(\alpha-1/e)$; $\alpha = U_2/U_1 = \rho_1/\rho_2$; $U_3\tau = L$ and the velocities in the 1st and 3rd regions are obtained as follows:

$$v'_1 = v'_{11};\quad v'_3 = \sum_{k=1}^{2} v'_{3k}.$$

"The sewing together" of the disturbed states in the 1st and 3rd regions is carried out using the conservation of mass law in its linearized form:

$$\left(v'_3+\frac{p'_3}{\rho_3 U_3}M_3^2-\frac{U_3}{c_{p3}}S'_3\right)_{x=-L} = q\left(v'_1+\frac{p'_1}{\rho_1 U_1}M_1^2\right)_{x=-L}. \tag{3}$$

In this case the law of mass conservation is identical with a similar law in standard gas dynamics [1, 2]. Moreover, the conservation law of impulse is also used. One linearizes it and one obtains

$$\frac{p'_3}{\rho_3 U_3}(1+M_3^2)-\frac{U_3}{c_{p3}}S'_3+\frac{M_{m3}^2}{M_3^2}\cdot\frac{h'_3 U_3}{H_0}+2v'_3 = 2v'_1+\frac{p'_1}{\rho_1 U_1}(1+M_1^2). \tag{4}$$

The conservation of energy law in its linearized form applied to disruption plane x = –L leads to the following expression:

$$v'_3+\frac{p'_3}{\rho_3 U_3}(1-M_{m3}^2)+\frac{S'_3}{c_{p3}}\cdot\frac{U_3}{M_3^2}\left(\frac{1}{\varkappa_3-1}-M_{m3}^2\right)+2\frac{h'_3 U_3}{H_0}\left(\frac{M_{m3}}{M_3}\right)^2 = \frac{1}{q}\left[v'_1+\frac{p'_1}{\rho_1 U_1}\right]. \tag{5}$$

The joining together of the disturbed states in the 1st and 2nd regions is obtained by using the continuity laws of the flow of mass, of impulse and of energy during the passage through the flame. These laws can easily be expressed in the same way as the formulas (3)-(5). In the equations of mass and energy flow one should take the relative gas velocity relative to the flame instead of the disturbance velocity v'_j, that is, $v'_j - d\varepsilon/dt$. Bearing this in mind we have:

a) the conservation law of the mass flow

$$\alpha\left[v_1' - \frac{d\varepsilon}{dt} + \frac{p_1'}{\rho_1 U_1}M_1^2\right]_{x=-L} = \left[v_2' - \frac{d\varepsilon}{dt} + \frac{p_2'}{\rho_2 U_2}M_2^2 - \frac{U_2}{c_{p2}}S_2'\right]_{x=0}; \tag{6}$$

b) the conservation law of the impulse flow

$$\left[2v_1' + \frac{p_1'}{\rho_1 U_1}(1+M_1^2)\right]_{x=-L} = \left[2v_2' + \frac{p_2'}{\rho_2 U_2}(1+M_2^2) - \frac{U_2}{c_{p2}}S_2' + \frac{M_{m2}^2}{M_2^2}\cdot\frac{h_2' U_2}{H_0}\right]_{x=0}; \tag{7}$$

c) the conservation law of the energy flow

$$\frac{1}{\alpha}\left[v_1' - \frac{d\varepsilon}{dt} + \frac{p_1'}{\rho_1 U_1}\right]_{x=-L} = \left[v_2' + \frac{p_2'}{\rho_2 U_2}(1 - M_{m2}) + \frac{S_2^1}{c_{p2}}\cdot\frac{1}{M_2^2}\left(\frac{1}{\varkappa_2 - 1} - M_{m2}^2\right) + 2\frac{h_2' U_2}{H_0}\left(\frac{M_{m2}}{M_2}\right)^2\right]_{x=0}. \tag{8}$$

We have thus obtained the homogeneous system (2), (3)-(8) to determine the function $\omega = f(M_j, M_{mj}, U_j)$. One sets the determinant of the system equal to zero so that all the constants appearing in the solution, namely A_{11}, A_{22}, A_{31}, A_{32}, B, D_2, D_3, be different from zero. This leads to the following characteristic equation

$$\frac{1}{\beta_2}(\Pi_2 + R_2) - R_2 N_2 + (\alpha - 1)\left[R_4 - \varphi_2\beta_3 R_1 - \frac{\varphi_1 - \varphi_2}{2N_3}(\Pi_1 - R_1)\right]\left[N_2\left(1 + \frac{1}{\beta_2}\right) + \frac{1}{\beta_2}\left(\frac{1}{\alpha} - 1\right)\right] = 0,$$

where

$$R_1 = \frac{1}{\beta_1} - 1 + q\left(\frac{1}{\beta_3} + 1\right);\ R_2 = 1 - \frac{1}{M_1} - \alpha\left(1 + \frac{1}{\beta_2}\right);$$

$$R_3 = q\varphi_2 - \frac{1}{1 - M_1};\ R_4 = \frac{1}{2}\beta_3 R_1(\varphi_1 - \varphi_2) - R_3;\ \beta_j = \frac{M_j}{\sqrt{1 + M_{mj}^2}};$$

$$N_j = \frac{M_{mj}^2 - (\varkappa_j - 1)^{-1}}{M_j^2}\quad (j = 2,3);\ \Pi_1 = \frac{1}{\beta_1 q} + \frac{q}{\beta_3};$$

$$\varphi_k = \beta_3\left[\exp(\gamma_{3k}L) - \exp\left(-\frac{z}{q}\right)\right][1 + (-1)^k\beta_3]^{-1};\ z = \frac{\omega L}{U_1};$$

$$\gamma_{3k}L = \frac{z}{q}\,\frac{\beta_3}{(-1)^{k+1} + \beta_3};\ \Pi_2 = \frac{1}{\alpha M_1} + \frac{\alpha}{\beta_2};\ \beta_1 \equiv M_1.$$

If $M_j \ll 1$ (incompressible gas) or $M_{mj} \gg 1$ (a strong magnetic field) then the equation simplifies, becoming

$$(R_1 - \varphi_2\beta_3 R_3)(\alpha - 1)\left(1 + \frac{1}{\alpha}\right) - R_2 = 0. \tag{9}$$

Expanding $\exp(\gamma_{3k}L)$ into a power series and retaining only terms up to the order $O(z\beta_3^2)$ we find from (9)

$$\exp\left(-\frac{z}{q}\right) = \frac{\beta_2 + \alpha M_1}{(\alpha - 1)(2qM_1 + \beta_3)\beta_3} \backsim \frac{1}{\beta_3} = \frac{\sqrt{1 + M_{m3}^2}}{M_3}. \tag{10}$$

If $M_{m3} \to 0$ then the well-known result follows as regards the one dimensional stability in standard gas dynamics [1]. If one compares it with standard gas dynamics, one can see that in the case under consideration one can have in accordance with [1] $\mathrm{Re}\,z < 0$ ($\mathrm{Re}\,\omega < 0$), and M_j need not necessarily be very small. Thus one may be able in this case to achieve stability by means of an external factor, namely the magnetic field. Then

$$z = -q\{\ln\sqrt{1 + M_{m3}^2} - \ln M_3\} < 0;\ M_m > M; \tag{11}$$

the latter enables one to extend the stability zone of combustion compared with the nonmagnetic case.

Indeed, if similarly as in [1, 2] one assumes $\omega \sim 1/\lambda$ (the disturbance intensity decreases with the increasing wavelength) then according to (11) the wavelength of unstable disturbances in the case under consideration, or more exactly the value λ/L need not be a very large quantity. This is due to the fact that such disturbances are damped by the magnetic field.

NOTATION

p is the pressure;
v is the velocity;
ρ is the density;
S is the entropy;
$\varkappa$ is the heat capacity ratio;
c is the sound velocity;
c_p is the heat capacity;
ω is the frequency (eigenvalue);
L is the width of flame front;
H_0 is the magnetic field intensity;
λ is the wavelength of flame disturbance for two dimensional case;
ε is the flame front displacement.

Primed quantities refer to the disturbed state.

Subscripts

1, 2, 3 correspond to regions of reference mixture, of combustion products and of flame respectively;
M is the Mach number;
M_m is the "magnetic" Mach number;
$\alpha = U_2/U_1$;
$q = U_3/U_1$;
$\beta_j = M_j/\sqrt{1 + M_{mj}^2}$.

LITERATURE CITED

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